Take Some Time to Reflect

Reflections of Linear and Exponential Functions

5.4

LEARNING GOALS

In this lesson, you will:

- Reflect linear and exponential functions vertically.
- Reflect linear and exponential functions horizontally.
- Determine characteristics of graphs after transformations.

KEY TERMS

- reflection
- line of reflection

You are already familiar with many different types of "reflections" in mathematics. When a negative sign is present, this is a good indication of a reflection.

For example, -5 is a "reflection" of 5 over 0 on the number line.

The power $2^1 = \frac{2}{1}$, but $2^{-1} = \frac{1}{2}$.

In this lesson, you will learn about reflecting functions. Watch out for those negative signs!

PROBLEM 1 Reflections

Consider the three exponential functions shown, where $h(x) = 2^x$ is the basic function.

- $h(x) = 2^x$
- $m(x) = -(2^x)$
- $n(x) = 2^{(-x)}$

1. Write the functions $m(x)$ and $n(x)$ in terms of the basic function $h(x)$.

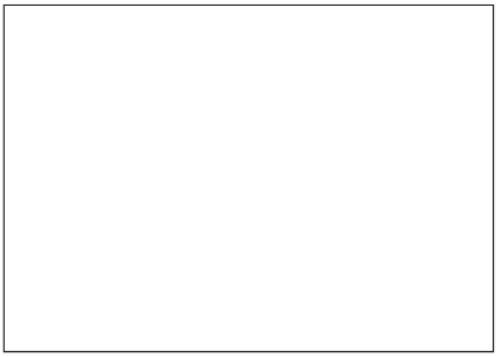
   $m(x) =$ _____

   $n(x) =$ _____

2. Compare $m(x)$ to $h(x)$. Does an operation performed on $h(x)$ or on the argument of $h(x)$ result in the equation for $m(x)$? What is the operation?

3. Compare $n(x)$ to $h(x)$. Does an operation performed on $h(x)$ or on the argument of $h(x)$ result in the equation for $n(x)$? What is the operation?

4. Use a graphing calculator to graph each function with the bounds $[-10, 10] \times [-10, 10]$. Then, sketch the graph of each function. Label each graph.



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5. Compare the graphs of $m(x)$ and $n(x)$ to the graph of the basic function $h(x)$. What do you notice?

6. Complete the table of ordered pairs for the three given functions.

$h(x) = 2^x$	$m(x) = -(2^x)$	$n(x) = 2^{(-x)}$
$(-2, \frac{1}{4})$	$(-2, \_\_\_\_)$	$(\_\_\_\_, \frac{1}{4})$
$(-1, \frac{1}{2})$	$(-1, \_\_\_\_)$	$(\_\_\_\_, \frac{1}{2})$
(0, 1)	$(0, \_\_\_\_)$	$(\_\_\_\_, 1)$
(1, 2)	$(1, \_\_\_\_)$	$(\_\_\_\_, 2)$
(2, 4)	$(2, \_\_\_\_)$	$(\_\_\_\_, 4)$

7. Use the table to compare the ordered pairs of the graphs of $m(x)$ and $n(x)$ to the ordered pairs of the graph of the basic function $h(x)$. What do you notice?

A **reflection** of a graph is a mirror image of the graph about a *line of reflection*. A **line of reflection** is the line that the graph is reflected about. A horizontal line of reflection affects the y-coordinates, and a vertical line of reflection affects the x-coordinates.

You can use the coordinate notation shown to indicate a reflection about a horizontal line of reflection.

$$(x, y) \rightarrow (x, -y)$$

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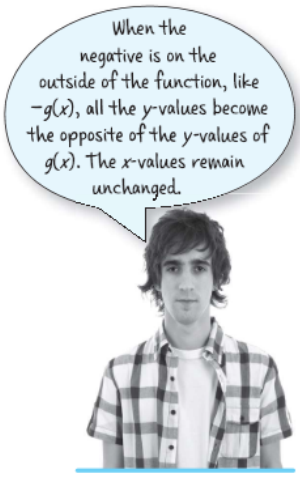
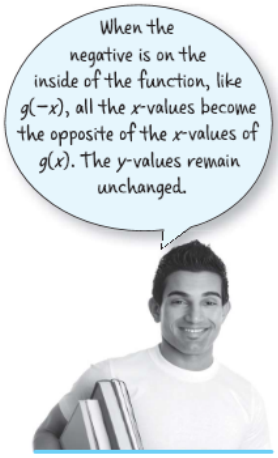


You can also use the coordinate notation shown to indicate a reflection about a vertical line of reflection.

$$(x, y) \rightarrow (-x, y)$$



8. Which function represents a reflection of $h(x)$ over a horizontal line? Which function represents a reflection of $h(x)$ over a vertical line?

9. Describe each graph in relation to its basic function.

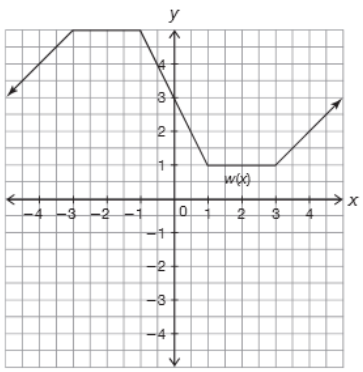
a. Compare $f(x) = -(b^x)$ to the basic function $h(x) = b^x$.

b. Compare $f(x) = b^{(-x)}$ to the basic function $h(x) = b^x$.

10. The graph of a function $w(x)$ is shown. Sketch the graphs of $w'(x)$ and $w''(x)$.

a. $w'(x) = -w(x)$

b. $w''(x) = w(-x)$



11. Write the equation of each function after a reflection about the horizontal line $y = 0$. Then, write the equation after a reflection about the vertical line $x = 0$.

a. $a(x) = 5^x$

Reflection about $y = 0$: $a'(x) =$ _____

Reflection about $x = 0$: $a''(x) =$ _____

b. $b(x) = -2x^2$

Reflection about $y = 0$: $b'(x)$ _____

Reflection about $x = 0$: $b''(x)$ – _____

c. $c(x) = \frac{5}{4}x^3$

Reflection about $y = 0$: $c'(x)$ = _____

Reflection about $x = 0$: $c''(x)$ = _____

PROBLEM 2 Linear Functions . . . Another Curious Case!

Consider the three linear functions shown, where $g(x) = x$ is the basic function.

- $g(x) = x$
- $r(x) = -(x)$
- $s(x) = (-x)$

1. Write the functions $r(x)$ and $s(x)$ in terms of the basic function $g(x)$.

$r(x)$ = _____

$s(x)$ – _____

2. Use a graphing calculator to graph each function with the bounds $[-10, 10] \times [-10, 10]$. Then, sketch the graph of each function. Label each graph.

3. Compare the graphs of $r(x)$ and $s(x)$ to the graph of the basic function $g(x)$. What do you notice?

4. Complete the table of ordered pairs for the three given functions.

$g(x) = x$	$r(x) = -(x)$	$s(x) = (-x)$
(−2, −2)	(−2, ____)	(____, −2)
(−1, −1)	(−1, ____)	(____, −1)
(0, 0)	(0, ____)	(____, 0)
(1, 1)	(1, ____)	(____, 1)
(2, 2)	(2, ____)	(____, 2)

5. Use the table to compare the ordered pairs of the graphs of $r(x)$ and $s(x)$ to the ordered pairs of the graph of the basic function $g(x)$. What do you notice?

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6. Which function represents a reflection of the basic function $g(x)$ over a vertical line? Which function represents a reflection of the basic function $g(x)$ over a horizontal line? Why do you think they produce the same graph?

PROBLEM 3 Characteristics of Graphs after Transformations

1. Use the given characteristics to write an equation and then sketch a graph of $g(x)$.

   a. Write an equation and sketch a graph that:

   - is an exponential function,
   - is continuous,
   - is increasing, and
   - is translated 2 units to the left of $f(x) = 2^x$.

   Equation: $g(x) =$ _____

   b. Write an equation and sketch a graph that:

   - is an exponential function,
   - is continuous,
   - is increasing, and
   - is translated 5 units down from $f(x) = 2^x$.

   Equation: $g(x) =$ _____

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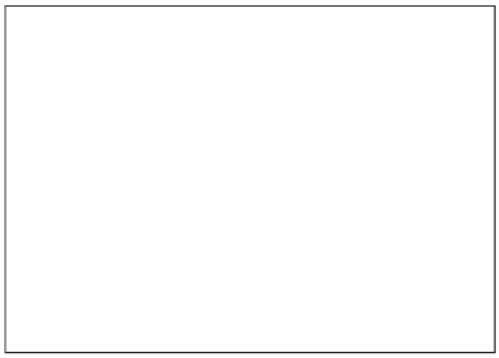
c. Write an equation and sketch a graph that:

- is an exponential function,
- is continuous,
- is decreasing, and
- is a reflection of $f(x) = 2^x$ over the line $x = 0$.

Equation: $g(x) =$ _____



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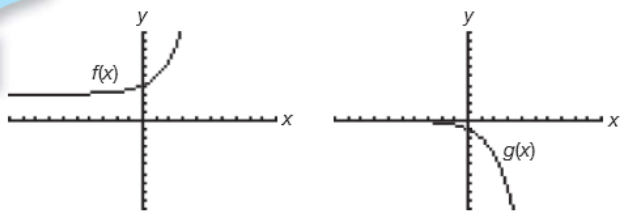
2. Jacob and Kate are comparing the two graphs shown.



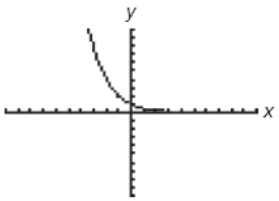
Jacob says that to get the graph of $g(x)$, first translate $f(x)$ down 3 units, and then reflect over the line $y = 0$. Kate says that to get the graph of $f(x)$, first reflect $g(x)$ over the line $y = 0$, and then translate up 3 units.

Who is correct? Explain your reasoning.

3. Choose the transformations in the box performed on $f(x)$ that would produce the graph of $g(x)$. Then sketch the graph of $g(x)$ on the coordinate plane.

vertical translation	horizontal translation
reflection over the line $y = 0$	reflection over the line $x = 0$

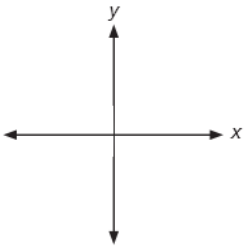
a. The graph of $f(x)$ is shown.



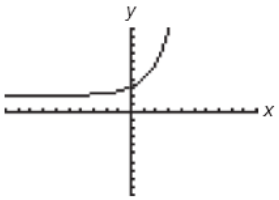
The graph of $g(x)$ has a different y-intercept than the graph of $f(x)$, but the same asymptote as $f(x)$.

Possible transformation(s) on $f(x)$ to produce $g(x)$:

Sketch of $g(x)$



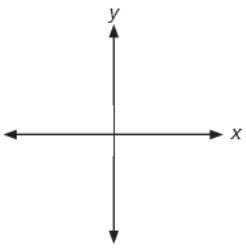
b. The graph of $f(x)$ is shown.



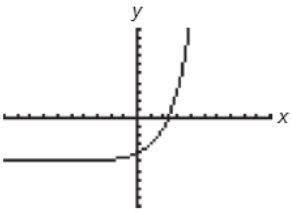
The graph of $g(x)$ has the same y-intercept as $f(x)$, but $g(x)$ is decreasing.

Possible transformation(s) on $f(x)$ to produce $g(x)$:

Sketch of $g(x)$



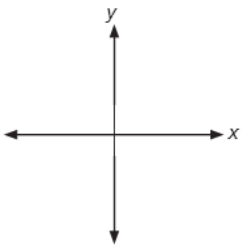
c. The graph of $f(x)$ is shown.



The graph of $g(x)$ has a different asymptote than the graph of $f(x)$, and $g(x)$ is increasing.

Possible transformation(s) on $f(x)$ to produce $g(x)$:

Sketch of $g(x)$



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Be prepared to share your solutions and methods.